Minimum Achievable Phase Noise of *RC* Oscillators

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Abstract—To make *RC* oscillators suitable for RF applications, their typically poor phase-noise characteristics must be improved. We show that, for a given power consumption, this improvement is fundamentally limited by the fluctuation-dissipation theorem of thermodynamics. We also present the analytical formulation of this limit for relaxation (including ring) oscillators using a time-domain phase-noise analysis method which is introduced in this paper. Measurement shows the maximum possible improvement is generally less than 6 dB for ring oscillators, while it can be as high as 21 dB for other relaxation oscillators. The suboptimal performance of relaxation oscillators is attributed to the continuous current flow in these oscillator topologies. These results provide useful insight for feasibility studies of oscillator design.

Index Terms—Integrated oscillator, jitter, phase noise, relaxation oscillator, ring oscillator.

NOMENCLATURE

C	Capacitor.
e	$= 2.7182818$
f	Frequency.
Δf	Offset frequency.
f_o	Nominal oscillation frequency.
g_{do}	MOS output conductance for $v_{ds} = 0$.
g_o	MOS output conductance in saturation.
I_C	Inverter current in *on* state (ring oscillators).
i_n	Current noise source.
k	Boltzmann constant.
L	Inductor.
l	Length of a transmission line.
$L_{\min}$	Length of the shortest transistor in circuit (ring oscillators).
M	Number of oscillators in a coupled-oscillator system.
N	Number of inverters (ring oscillators).
$P_{\min}$	Minimum power dissipation.
PN	Phase noise.
PN_{meas}	Measured phase noise.
$PN_{\min}$	Minimum phase noise for a given power.
ΔPN	Wastefulness factor, $PN_{\text{meas}} - PN_{\min}$.
R	Resistor.
R_n	Equivalent noise resistor.
T	Temperature.
t	Time.
T_o	Nominal oscillation period.
ΔT_o	Variation of the period relative to the nominal value (the variance of this random variable is defined as period jitter).
T_1, T_2	Nominal duration of first and second halves of the period.
$\Delta T_1, \Delta T_2$	Variations of the durations of the first and second half of the period with respect to the nominal values.
T_E	Time spent in equilibrium (ring oscillators).
T_s	Switching time.
ΔT_s	Variation of the switching time relative to the nominal time (switching time jitter).
T_{T1}, T_{T2}	Transition times (ring oscillators).
v	Electromagnetic wave velocity in a transmission line.
v_1, v_2	Decision levels for a switch or comparator.
v_{1n}	Normalized decision level, v_1/v_{dd}.
v_C	Capacitor voltage.
Δv_C	Variation of the capacitor voltage relative to the nominal value.
σ_o^2	Variance of Δv_c at $t = 0$.
$v_{\text{A,B,C}}$	Voltage at node A, B, or C.
v_{dd}	Supply voltage.
v_{ds}	MOS drain-to-source voltage.
v_T	MOS threshold voltage.

I. INTRODUCTION

THE exponential growth of the wireless communications industry has augmented the demand for more communication channels. This demand is usually hard to satisfy because the number of available channels is limited by the receiver's capability to accurately tune to a specific channel without being affected by the adjacent interfering channels. As shown in Fig. 1, an interfering signal can modulate the phase noise of the local oscillator and generate an interfering tone at the intermediate frequency. Thereafter, this interfering tone will be indistinguishable from the desired signal and thus cannot be removed by filtering. Since the phase noise of the local oscillator determines the power of this interfering tone, it is of great practical significance.

The significance of phase noise in RF systems limits the usefulness of *RC* oscillators[1] because of their typically inferior phase noise properties compared to inductor-based and distributed oscillators. RF designers need to improve these properties in order to benefit from the attractive integrated nature of *RC* oscillators, a virtue that has made them popular for clock-recovery circuits [1] and on-chip clock distribution [2]. Several investigations have focused on improving the frequency

[1] In this paper, lumped, inductorless oscillators, including ring and other relaxation oscillators, are all referred to as *RC* oscillators.
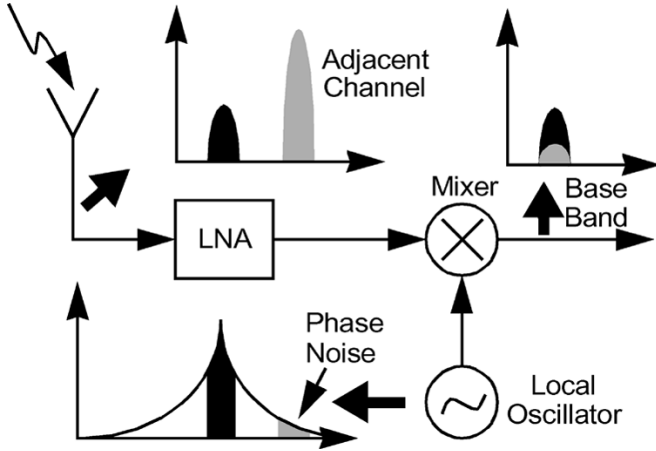
Fig. 1. Effect of phase noise on RF systems, a simplified diagram.

stability of *RC* oscillators (e.g., [3]). However, the literature does not include a study of the possible theoretical limit on the phase noise of *RC* oscillators for a given power. Such a study will provide pre-design knowledge about the feasibility of designing low-phase-noise *RC* oscillators and will reduce the lingering uncertainty about the future of RFIC design.

In this paper, we extend our results presented in [4] and show that one of the fundamental principles of thermodynamics, the fluctuation-dissipation theorem, sets a lower limit on the phase noise of *RC* oscillators. This limit is a function of the power dissipation, temperature, frequency of oscillation and offset frequency at which we measure phase noise. In order to calculate this lower limit, we first introduce a phase-noise analysis method based on time-domain jitter analysis. We then use this method to calculate the minimum achievable phase noise for relaxation (including ring) oscillators. Phase noise in coupled *RC* oscillators is also discussed. Although some parts of the presented work have been briefly reported in [4], our work on ring oscillators (Section III-C) and coupled oscillators (Section III-D) are presented for the first time here. Finally, we present several examples of relaxation oscillators to compare the minimum achievable phase noise to the measurement results followed by a discussion of the implications of these results. This discussion is intended to help designers identify performance-limiting mechanisms in *RC* oscillators.

II. PHYSICAL ARGUMENT FOR MINIMUM ACHIEVABLE PHASE NOISE

Nonzero phase noise in an oscillatory signal indicates that the period of oscillation is not truly constant. To stabilize this period (and hence to build a low-phase-noise oscillator), we should make the period of oscillation dependent on a reliable physical phenomenon, one which can force the oscillation period to be traceable to a physical constant with the dimension of time. In inductor-based oscillators (like the Colpitts) this constant is $\sqrt{LC}$ where L is the inductor and C is the capacitor. In transmission-line-based oscillators the ratio of l/v establishes the time constant in which l is the length of the transmission line and v is the velocity of electromagnetic wave inside the transmission line. In *RC* oscillators the product of *RC* is usually the time constant. There is, however, a fundamental difference between

this latter case and the first two cases. The fluctuation-dissipation theorem of thermodynamics states that a nonzero amount of thermal noise is associated with any resistor.[2] Thus, in contrast to lossless-inductor-based and lossless-transmission-line-based oscillators, the time constant of *RC* oscillators is inherently noisy because of the resistor dissipation. This noise component can affect the period of oscillation in a random fashion and result in nonzero phase noise. Consequently, even if the rest of the circuit is noise-free, the resistor noise imposes a lower limit on the phase noise of *RC* oscillators.

To provide a quantitative prediction of this minimum achievable phase noise, we use simple models for relaxation oscillators. For the formulation of minimum achievable phase noise of relaxation oscillators, only the equilibrium noise current of the resistor (given by $4kT/R$) is taken into account. For the special case of ring oscillators, we assume that the only noise sources in the circuit are the noise sources associated to the main transistors. The power spectral density of this noise source is predicted by the long-channel MOSFET noise theory. In order to provide a quantitative analysis of minimum achievable phase noise, we first introduce a time-domain phase-noise analysis method for switching-based oscillators. We then apply this method to relaxation oscillators.

III. ANALYTICAL FORMULATION OF PHASE NOISE

A. Time-Domain Phase Noise Analysis for Switching-Based Oscillators

In switching-based oscillators the energy injecting elements act like ideal switches, i.e., they have a countable number of states, between which transitions may be considered instantaneous. Relaxation oscillators can be modeled as switching based oscillators. For this class of oscillators, phase noise is most easily calculated by first calculating the jitter in the time domain. Since the oscillator's feedback path is broken by the active device during the nonswitching time interval, the calculation of jitter for these oscillators is relatively simple. Once the jitter is determined, phase noise can be calculated using available mathematical relationships. In the rest of this subsection we elaborate on how these calculations are performed.

Fig. 2(a) shows part of a switching based oscillator. The energy injecting element switches when the voltage of the control terminal reaches v_1. We use the first crossing approximation, which assumes that the switching takes place when the voltage reaches v_1 for the first time [5]. Using a linear approximation, the variance of the switching time jitter is found to be proportional to the variance of the control terminal voltage and inversely proportional to the square of the rate of change of voltage at this node.

The variance of the control terminal voltage can be calculated by knowing the total resistance and total capacitance on this node as well as the stochastic properties of the noise sources connected to this node. Fig. 2(b) shows the noise circuit model

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[2]The fluctuation dissipation theorem is strictly applicable to the thermal equilibrium state of the resistor. Nonetheless, experimental observations show that a resistor's nonequilibrium noise is also finite and, in most cases, has the same value as that of the thermal equilibrium noise. The power spectral density of this noise source is given by Nyquist formula, a formula that will be used for quantitative analysis in this paper.
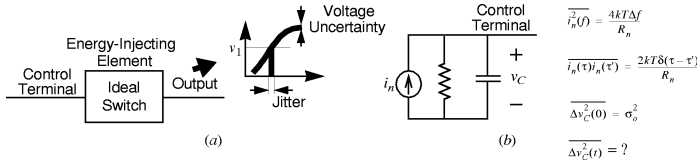
Fig. 2. (a) Switching time jitter in switching-based oscillator. (b) Noise circuit model and problem definition for calculation of voltage uncertainty on the control terminal.

which is used for these calculations. The problem definition is also given in the inset of this figure. The variance of the capacitor voltage at $t = 0$ is assumed to be a Gaussian random variable with zero mean and a variance of σ_o^2. We are interested in calculating the variance of this voltage at some later time t. The voltage on the capacitor at time t is given by

$$\Delta v_C(t) = \frac{e^{-t/RC}}{C} \int_0^t e^{\tau/RC} i_n(\tau) d\tau + \Delta v_C(0) e^{-t/RC}. \quad (1)$$

Since i_n is a Gaussian process with zero mean, (1) dictates that Δv_C is also a Gaussian process with zero mean [6]. Consequently, the fluctuation properties of Δv_C are completely conveyed by its variance. Using (1) we can find the variance of Δv_C at time t

$$\overline{\Delta v_C^2(t)} = \frac{e^{-2t/RC}}{C^2}$$
$$\times \int_0^t \int_0^t e^{(\tau+\tau')/RC} \overline{i_n(\tau) i_n(\tau')} d\tau d\tau' + \sigma_o^2 e^{-2t/RC} \quad (2)$$

in which $\overline{i_n(\tau) i_n(\tau')}$ is the autocorrelation function of the noise source. In this paper we assume that the noise source is white with the autocorrelation function given in the onset of Fig. 2(b). Using this autocorrelation function in (2) and performing the integration, the variance of Δv_C is found to be[3]

$$\overline{\Delta v_C^2(t)} = \frac{kTR}{CR_n} \left(1 - e^{-2t/RC}\right) + \sigma_o^2 e^{-2t/RC}. \quad (3)$$

Note that if $R_n = R$ the variance of v_C converges to kT/C as $t \to \infty$ and becomes independent of R and σ_o^2.

Under the linear approximation, the variance of the switching time jitter is simply the variance of Δv_C divided by the square of the capacitor's voltage rate of change

$$\overline{\Delta T_s^2} = \overline{\Delta v_C^2(t)} \left| \frac{\partial v_C}{\partial t} \right|^{-2}. \quad (4)$$

The total period jitter in the oscillator is then calculated by adding up all individual switch time jitters in the oscillator which are assumed to be independent in the absence of colored noise.

Once the period jitter is calculated, phase noise can easily be calculated. In most cases (including relaxation oscillators) the output of the switching oscillator can be approximated by a stochastic square wave signal with mutually independent, Gaussian-distribution period jitter. As presented in [7] and [8], the phase noise of such a signal has a nearly Lorentzian

[3]The presented derivation is simplified for brevity. A rigorous derivation shows that the final result is, nevertheless, correct.
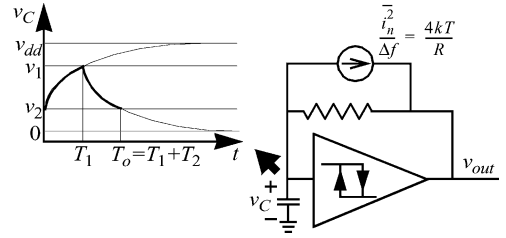

Fig. 3. Typical relaxation oscillator and the respective wave form.

shape around each harmonic.[4] The phase noise around the first harmonic at an offset frequency of Δf is given by

$$PN(\Delta f) = \frac{f_o^3 \overline{(\Delta T_o)^2}}{(\pi f_o^3 \overline{(\Delta T_o)^2})^2 + (\Delta f)^2} \quad (5)$$

where f_o and Δf are center frequency and offset frequency, respectively, and $\overline{(\Delta T_o)^2}$ is the variance of the period. This equation predicts that the phase noise has a $1/f^2$ shape for sufficiently large offset frequencies, which is consistent with the previously reported measurement results [9], [10].

Note that (5) is valid only if period jitters of different cycles are mutually independent. In the presence of colored noise this condition is usually violated and hence (5) would lose its validity. In this paper we do not consider the effect of colored noise on phase noise.

B. Phase Noise in RC Relaxation Oscillators

Fig. 3 shows a typical *RC* relaxation oscillator. The oscillator is composed of a Schmitt comparator in an *RC* feedback loop. We first derive the basic equations governing the behavior of this oscillator and then present an analysis for jitter and phase noise. Since we are interested in the minimum achievable phase noise, we take into account only the equilibrium resistor noise (whose density is given by $4kT/R$) and neglect all other noise sources associated with the comparator and all nonequilibrium noise sources (like $1/f$ noise).

The oscillator of Fig. 3 works as follows: during the first half of the period, the capacitor voltage changes exponentially from v_1 to v_2 (the two comparison levels). The duration of the first half of the period is found to be

$$T_1 = RC \times \ln\left(\frac{v_{dd} - v_1}{v_{dd} - v_2}\right). \quad (6)$$

Similarly, the duration of the second half of the period is

$$T_2 = RC \times \ln\left(\frac{v_2}{v_1}\right) \quad (7)$$

and the frequency of oscillation is given by

$$f_o = \frac{1}{T_o} = \frac{1}{T_1 + T_2} = \frac{1}{RC \times \ln\left(\frac{v_{dd}-v_1}{v_{dd}-v_2} \times \frac{v_2}{v_1}\right)} \quad (8)$$

where T_o is the nominal period of oscillation.

The absolute minimum power dissipation of this oscillator (neglecting the power consumed by the comparator) is dictated

[4]In this paper, phase noise is defined as the single-sided power spectral density of the signal normalized to the power in the fundamental tone.

by the amount of charge transferred to the capacitor as its voltage moves between v_1 and v_2 in each cycle:

$$P_{\min} = v_{dd}C(v_2 - v_1)f_o. \tag{9}$$

The nonzero minimum power dissipation is another distinct property of *RC* oscillators. For lossless *LC* and transmission-line-based oscillators, the minimum power dissipation is zero; the energy can be exchanged losslessly between two energy-storage elements.

The jitter is generated by the fluctuations of v_C which are in turn caused by the resistor noise. The probability density function of the fluctuations of v_C at any given time t can be found using the circuit model of Fig. 2(b). Since the switching takes place when the capacitor voltage reaches the decision levels (v_1 and v_2), this voltage is accurately known at the beginning of each half-period. Using (3) with $\sigma_o^2 = 0$ and $R_n = R$ we get

$$\overline{\Delta v_C^2(t)} = \frac{kT}{C}\left(1 - e^{-2t/RC}\right). \tag{10}$$

Using $t = T_1$ or T_2 in this equation, we can find the variance of Δv_C at the end of the first and second half-periods, respectively.

The switching time jitter at the end of each half period can be found using (4)

$$\overline{\Delta T_1^2} = \left|\frac{\partial v_C}{\partial t}\right|_{t=T_1}^{-2}\overline{\Delta v_C^2} = \left(\frac{RC}{v_{dd}-v_2}\right)^2\overline{\Delta v_C^2} \tag{11}$$

$$\overline{\Delta T_2^2} = \left|\frac{\partial v_C}{\partial t}\right|_{t=T_o}^{-2}\overline{\Delta v_C^2} = \left(\frac{RC}{v_1}\right)^2\overline{\Delta v_C^2} \tag{12}$$

where the linearization is justified due to the fact that the noise level is normally small. The variance of the period is the sum of (11) and (12) because ΔT_1 and ΔT_2 are uncorrelated variables based on the following argument. The switching takes place when v_C equals v_1 or v_2. Consequently, at the beginning of each half cycle the value of v_C is known accurately. Since the only noise source taken into account is white, the uncertainty of v_C at the end of each half cycle is completely independent of the one at the end of previous half cycle. This ensures that ΔT_1 and ΔT_2 are in fact independent and the variance of the period jitter is the sum of (11) and (12)

$$\overline{\Delta T_o^2} = kTR^2C\left(\frac{1}{(v_{dd}-v_2)^2} + \frac{1}{v_1^2}\right)\left(1 - e^{-T_o/RC}\right) \tag{13}$$

in which we have assumed a duty cycle of 50% so that we can replace $2t$ in (10) by T_o.

After eliminating R between (8) and (13) we get

$$\overline{\Delta T_o^2} = \frac{kT}{2Cf_o^2v_{dd}^2} \cdot \frac{(1 - 2v_{1n})}{v_{1n}^2\left[\ln\left(\frac{1}{v_{1n}} - 1\right)\right]^2(1 - v_{1n})^2} \tag{14}$$

where $v_{1n} = v_1/v_{dd}$ is the normalized decision level. We have assumed $v_2 = v_{dd} - v_1$, which ensures a duty cycle of 50%. The second part of (14), called the jitter factor, is only a function of v_{1n} and can be plotted versus this parameter. Fig. 4 presents such a plot and it shows that the jitter factor is minimized for $v_{1n} \approx 0.24$. For constant values of temperature, capacitance, oscillation frequency and bias voltage, jitter (and hence phase

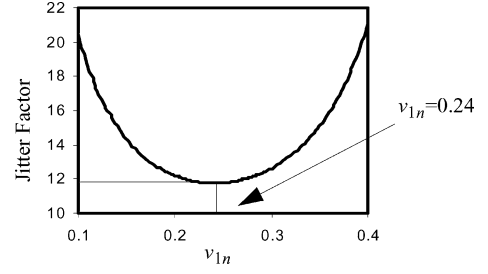

Fig. 4. Normalized jitter versus normalized decision level v_{1n}.

noise) assumes its minimum value for $v_1 \approx 0.24v_{dd}$ and $v_2 \approx 0.76v_{dd}$. Hereafter, we will use these values to find the minimum achievable phase noise. The minimum power dissipation and minimum period jitter are then found to be

$$P_{\min} \approx 0.52Cf_ov_{dd}^2 \tag{15}$$

$$\overline{\Delta T_o^2} = 2\overline{\Delta T_1^2} \approx \frac{5.9kT}{Cf_o^2v_{dd}^2} \approx \frac{3.1kT}{P_{\min}f_o}. \tag{16}$$

Using (16) in (5) the phase noise at offset frequencies much larger than $\pi f_o^3\overline{(\Delta T_o)^2}$, which are the only practically important frequencies, is found to be

$$PN_{\min}(\Delta f) \approx \frac{5.9f_okT}{Cv_{dd}^2(\Delta f)^2} \approx \frac{3.1kT}{P_{\min}}\left(\frac{f_o}{\Delta f}\right)^2. \tag{17}$$

It is instructive to compare this equation to the one presented in [11] for differential *LC* oscillators derived using Leeson's formula. This comparison shows that the phase noise of a differential *LC* oscillator is smaller than that of an *RC* oscillator roughly by a factor of Q^2. Equation (13) in [11] reduces to (17) with $Q = 1$, except for a numerical constant which is determined by oscillator's topology. This is also consistent with the discussion presented in [9] regarding definitions of Q in various oscillators.

Equation (17) also shows that the phase noise is inversely proportional to the minimum power dissipation, which is in turn proportional to the capacitor value and the square of the bias voltage. We will see that this result holds for ring oscillators as well.

C. Phase Noise in Ring Oscillators

To calculate the phase noise in ring oscillators, we model them as switching-based oscillators. In this model, each inverter is assumed to have only two distinctive states for noise calculations. In its *on* state (T_T), it delivers (or sinks) a constant current, I_C, independent of its input and output voltages. Its output resistance is $1/g_o$ in this state and its equivalent noise resistance is $3/(2g_{do})$, as predicted by the long-channel theory of MOSFET noise [12]. Here, g_o and g_{do} are the device output conductances in the saturation region and for zero v_{ds}, respectively. For simplicity, the two transistors in each inverter are assumed to be properly sized to be equivalent. In its *off* state (T_E), the inverter is in equilibrium and can be replaced by a resistor of $1/g_{do}$ because its output conductance is dominantly determined by the conducting device (Fig. 5).

In this model, the interstage noise effects are neglected. That is variations of the voltage on node A do not affect the voltage
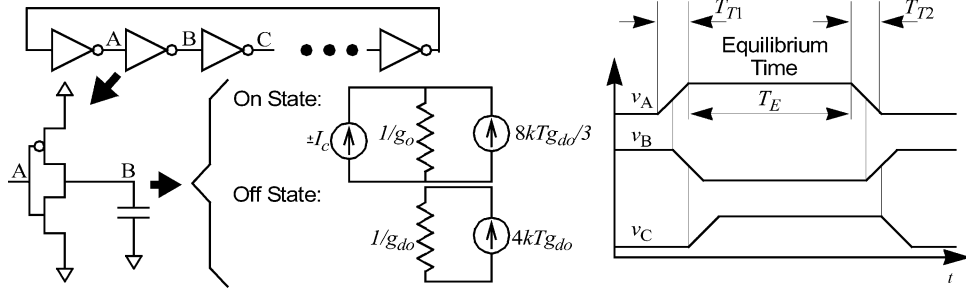
Fig. 5.   Ring oscillator modeled as a switching-based oscillator. Dominant noise sources are also demonstrated.

on node B regardless of the inverter's state. For simplicity we assume that the electrical properties of NMOS and PMOS transistors are identical. Furthermore, each inverter is assumed to switch when its input voltage reaches $v_{dd}/2$ (Fig. 5). These assumptions do not affect the general validity of the derivation.

To calculate phase noise one should first find the variance of the control voltage right before the switching moment. Since we have assumed that the switching takes place when the voltage reaches $v_{dd}/2$, the value of the control voltage is a deterministic variable after each switching. We can then use (3) with $\sigma_o^2 = 0$ repeatedly for various regions of one half-period, starting with the second half of T_{T1}, to find the variance of voltage at the new switching time. Fortunately certain simplifications are possible in the case of our model. According to (3), if the equilibrium time, T_E, is much longer than the time constant C/g_{do}, the variance of the control voltage at the end of this time interval is kT/C. This condition is usually well satisfied because g_{do} is a relatively large number. The approximation further improves for ring oscillators with many inverters.

During the first half of T_{T2}, the output impedance of the inverter is $1/g_o$ which is normally large. Consequently, we can assume that $T_{T2}/2$ is much smaller than C/g_o in this region and we can replace the exponential terms in (3) by their series expansion. The variance of control voltage at the switching time is then given by

$$\overline{\Delta v_C^2} = \frac{kT}{C}\left(1 + \frac{2T_{T2}g_{do}}{3C}\right) \tag{18}$$

where we have used $t = T_{T2}/2$, $\sigma_o^2 = kT/C$, $R_n = 2/(3g_{do})$, $R = 1/g_o$ and $t \ll RC$ in (3). Since we assumed that current is constant during the *on* state, (18) can be rewritten as

$$\overline{\Delta v_C^2} = \frac{kT}{C}\left(1 + \frac{2g_{do}}{3}\frac{v_{dd}}{I_C}\right) \tag{19}$$

and using (4) the variance of the switching time jitter is

$$\overline{\Delta T_s^2} = \frac{kTC}{I_C^2}\left(1 + \frac{2g_{do}}{3}\frac{v_{dd}}{I_C}\right). \tag{20}$$

Since there are $2N$ independent switching events in each period, the total period jitter will be

$$\overline{\Delta T_o^2} = \frac{2NkTC}{I_C^2}\left(1 + \frac{2}{3}\frac{g_{do}v_{dd}}{I_C}\right) \tag{21}$$

where N is the number of inverters in the oscillator. The nominal period of the signal from this oscillator is equal to the sum of all delays from individual inverters

$$T_o = \frac{NCv_{dd}}{I_C}. \tag{22}$$

After eliminating I_C between (21) and (22) and using the long-channel expression for g_{do} in (21), the period jitter is found to be

$$\overline{\Delta T_o^2} = \frac{2kTT_o^2}{NCv_{dd}^2}\left(1 + \frac{4}{3}\frac{v_{dd}}{v_{gs} - v_T}\right). \tag{23}$$

For calculation of the minimum phase noise for ring oscillators, we assume the optimum case of $v_{gs} \approx v_{dd}$, $v_T \approx v_{dd}/2$ and note that $P_{\min} = f_o NC v_{dd}^2$. We can then find the minimum jitter and minimum phase noise just as we did for other relaxation oscillators

$$\overline{\Delta T_o^2} \approx \frac{7.33kT}{NCv_{dd}^2 f_o^2} = \frac{7.33kT}{P_{\min} f_o} \tag{24}$$

$$PN_{\min}(\Delta f) \approx \frac{7.33 f_o kT}{NCv_{dd}^2(\Delta f)^2} = \frac{7.33kT}{P_{\min}}\left(\frac{f_o}{\Delta f}\right)^2. \tag{25}$$

D. Phase Noise in Coupled Oscillators

Coupled oscillators are used for several applications, including high-resolution delay circuits and multi-phase clock generators in digital systems [13]. Phase noise in these oscillator topologies (in their most general form) has been studied elsewhere and it is shown that the phase noise of a network of M optimally coupled, identical oscillators is M times smaller than that of each individual oscillator working by itself [14].

The time-domain analysis presented here supports this result. In a set of M coupled oscillators, if the random noise delays the transition at any of the nodes in any of the oscillators beyond the nominal transition time, the transitions in the corresponding node in other oscillators accelerate the transition through coupling. In an optimally coupled set of oscillators, the final jitter in each of the oscillators is the average of what the jitter would be in an individual oscillator if it were working by itself. Since the jitter in individual oscillators are independent random variables, the variance of their average is $1/M$ of the jitter in each single oscillator working by itself. This finding, combined with (5), dictates that the phase noise of a set of M optimally coupled oscillators is $1/M$ of the phase noise in each oscillator.
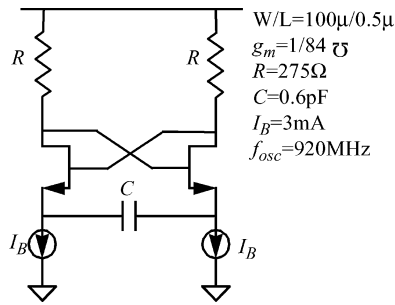
Fig. 6. Schematic and design parameters of the relaxation oscillator reported in [9].

Fig. 7. Minimum achievable phase noise compared to the data reported in [9] for an oscillator with $f_{\mathrm{osc}} = 920$ MHz and $P_{\min} = 19.8$ mW at $T = 300$ K.

The coupling of oscillators reduces the phase noise at the expense of higher power dissipation. A network of M coupled oscillators exhibits M times smaller phase noise compared to single oscillators while it also consumes M times more power. Equations (16) and (25) show that a suppression factor of M in the phase noise of individual oscillators is already achievable through increasing the power consumption by the same factor. The use of coupling is thus not particularly useful for phase noise suppression because it does not reduce the minimum achievable phase noise.

IV. EXPERIMENTAL RESULTS AND DISCUSSION

The expression given in (17) provides the minimum achievable phase-noise for the idealized version of the relaxation oscillator shown in Fig. 3. Most practical relaxation oscillators are not implemented exactly in this fashion. Fig. 6 provides the schematic and the design parameters of a CMOS relaxation oscillator reported in [9]. Although this oscillator is not exactly of the same form as the idealized relaxation oscillator given in Fig. 3, it can be modeled as such. In the case of the oscillator of Fig. 6, the charging and discharging mechanism of the capacitor is not through a resistor but rather through the current sources and the transistors. Nevertheless, these components are noisy and thus result in finite phase noise for this architecture. Comparing the measured phase noise of this oscillator with the minimum phase noise predicted by (17) for the same power gives a measure of the efficiency of the oscillator in terms of the power–phase-noise tradeoff. The difference between these two values is defined as the "wastefulness factor" in this paper.

Fig. 7 compares the phase noise reported in [9] to the minimum achievable phase noise given by the second part of (17) under a constraint of constant power. To calculate $P_{\min}$, we have assumed $v_{dd} = 3.3$ V, which is typical for the 0.5-μm technology. The minimum achievable phase noise for this power level is -122.6 dBc/Hz and -136.6 dBc/Hz at 1-MHz and 5-MHz offset frequencies, respectively. The measured values reported in [9] are -102 dBc/Hz and -115 dBc/Hz. The wastefulness factor is around 21 dB for this oscillator at these offset frequencies.

A similar architecture is reported in [10] as a relaxation VCO, which draws 2.3 mA of current from a 6-V power supply at 115 MHz ([10, Figs. 5(a) and 7]). Under constant power, (17) predicts that the minimum achievable phase noise for this oscillator is -139 dBc/Hz at an offset frequency of 1 MHz. This
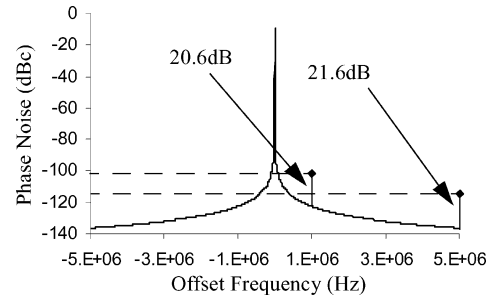
TABLE I
EXPERIMENTAL RESULTS VERSUS THEORETICAL PREDICTION OF MINIMUM ACHIEVABLE PHASE NOISE AT $\Delta f = 1$ MHz FOR THE RING OSCILLATORS OF [3]

Index	N	L_{min} μm	f_o MHz	Power mW	$PN_{meas.}$ dBc/Hz	PN_{min} dBc/Hz	ΔPN dB	Current Starved
1	5	2	232	1.5	-118.5	-119.7	1.2	No
2	11	2	115	2.5	-126	-128	2	No
4	3	0.53	751	5.85	-114	-115.4	1.4	Yes
5	5	0.39	850	6.27	-112.6	-114.6	2	Yes
6	7	0.36	931	6.22	-111.7	-113.8	2.1	Yes
7	9	0.32	932	6.82	-112.5	-114.2	1.7	Yes
8	11	0.32	869	6.62	-112.2	-114.6	2.4	Yes
9	15	0.28	929	7	-112.3	-114.3	2	Yes
10	17	0.25	898	9.5	-112	-115.9	3.9	Yes
11	19	0.25	959	9.75	-110.9	-115.5	4.6	Yes
3	19	0.25	1330	25	-111.5	-116.7	5.2	No

is again 21 dB lower than the reported value of -118 dBc/Hz given in [10]. These two examples illustrate that this particular relaxation oscillator configuration suffers a high wastefulness factor. The large wastefulness factor in these relaxation oscillators can be due to the continuous current flow in these oscillator topologies. It is also possible that the presence of other noise sources in the comparator lead to a high wastefulness factor.

Experimental data on ring oscillators shows that these oscillators have smaller wastefulness factors. Table I compares the measurement results reported in [3] to the theoretical prediction of the minimum achievable phase noise for the same power. The index numbers are the same as the ones assigned in [3]. N is the number of stages and $L_{\min}$ is the channel length of the shortest transistor in the circuit. The data is presented in the descending order of $L_{\min}$. Note that the simple equation given in (25) is capable of predicting the phase noise within a few decibels. This confirms the accuracy of the time-domain phase-noise calculation method presented in this paper.

Table I shows that the wastefulness factors of these ring oscillators are smaller than 6 dB with most numbers around 2 dB. This is much better than the relaxation oscillators discussed earlier. This table also shows that the wastefulness factor of a ring oscillator increases with decreasing $L_{\min}$. This phenomenon can be attributed to higher short-circuit switching current in faster transistors (when PMOS and NMOS transistors conduct

at the same time) or the higher excess noise in short-channel MOS devices. The origin of this second phenomenon is discussed in [15].

V. CONCLUSION

We have shown that the fluctuation-dissipation theorem of thermodynamics dictates a lower limit for phase noise in *RC* oscillators. To quantitatively predict this limit, we presented a time-domain phase-noise analysis method. Using this method, we calculated the minimum achievable phase noise for relaxation oscillators based on simple models. The results were expressed as functions of temperature, power dissipation, frequency of oscillation and the offset frequency. Our time-domain phase-noise analysis method provides a very simple formula which predicts the phase noise of ring oscillators within a few decibels.

We defined "wastefulness factor" as the difference between the minimum achievable phase noise and the actual measured phase noise for a specific oscillator. Using this definition, we showed that the wastefulness factor of ring oscillators is generally much smaller than that of other relaxation oscillators. Since the minimum achievable phase noise of ring oscillators is only 3.7 dB higher than that of other relaxation oscillators, we conclude that ring oscillators are more attractive in terms of power-phase-noise tradeoff. We also showed that, for practical ring oscillators, the phase noise improvement through careful design is fundamentally limited to 2–6 dB. The wastefulness factor of typical ring oscillators generally increases as we shrink the channel length of the MOS devices. This is attributed to the excess noise in short-channel MOS devices as well as higher short-circuit switching currents in smaller, faster devices. These findings provide guidelines for the efficient design of low-phase-noise *RC* oscillators.

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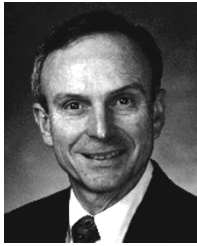
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